Day 1
Model and Process Uncertainty: Quantification and Usage

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Summary

Design vs. Operations/control problems

- Design: make decisions *once – have to get it right*
- Operations/control: make decisions iteratively – *we have recourse*

Operations/control problems

- Open-loop and closed-loop problems are very different problems
- Which factors impact (the most) the quality of implemented (online) solutions?
  Frequency of reoptimization, quality of model,
  quality of uncertainty representation, optimality gap, horizon?
- What can we do to obtain better solutions?
  Model uncertainty and solve multi-stage stochastic programming problem?
  Solve robust, adjustable robust optimization?
  What is a *good enough* planning horizon?
  Solve to optimality?
  Solve to suboptimal solutions but faster?

Design Problem

Example

- A manufacturing facility is used for the production of chemicals A and B
- The capacity of the facility is 10 tons/year;
  Can be expanded up to 12 tons/year at $100,000
- The demand for product A is expected to be constant: $D_A = 4$ ton/year
- The demand for product B is uncertain:
  Three scenarios (analysts): $D_B^{Low} = 4$, $D_B^{Med} = 6$ and $D_B^{High} = 8$ ton/year
- Profit from sales: Drug A = $100,000/ton & Drug B = $200,000/ton

Should we expand the facility?

Calculations using expected demand

✓ Expected demand: $D_A = 4$, $D_B = 6$ ton/year $\Rightarrow$ No expansion

✓ Expected profit:

- If $D_A = 4$, $D_B = 4$ ton/year $\Rightarrow P_A = 4$, $P_B = 4$ ton/year $\Rightarrow$ Profit = $1,200,000
- If $D_A = 4$, $D_B = 6$ ton/year $\Rightarrow P_A = 4$, $P_B = 6$ ton/year $\Rightarrow$ Profit = $1,600,000
- If $D_A = 4$, $D_B = 8$ ton/year $\Rightarrow P_A = 4$, $P_B = 6$ ton/year $\Rightarrow$ Profit = $1,600,000

✓ Objective = E[Profit] – Expansion Cost = **$1,466,667**

Design Problem

Example

- A manufacturing facility is used for the production of chemicals A and B
- The capacity of the facility is 10 tons/year;
  Can be expanded up to 12 tons/year at $100,000
- The demand for product A is expected to be constant: D_A = 4 ton/year
- The demand for product B is uncertain:
  Three scenarios (analysts): $D_B^{Low} = 4$, $D_B^{Med} = 6$ and $D_B^{High} = 8$ ton/year
- Profit from sales: Drug A = $100,000/ton & Drug B = $200,000/ton

Should we expand the facility?

Calculations using scenarios

✓ Consider expansion because demand can be for B can be as high as 8 ton/year

✓ Scenarios with expansion:

- If $D_A = 4$, $D_B = 4$ ton/year ⇒ $P_A = 4$, $P_B = 4$ ton/year ⇒ Profit = $1,200,000
- If $D_A = 4$, $D_B = 6$ ton/year ⇒ $P_A = 4$, $P_B = 6$ ton/year ⇒ Profit = $1,600,000
- If $D_A = 4$, $D_B = 8$ ton/year ⇒ $P_A = 4$, $P_B =$ **8** ton/year ⇒ Profit = **$2,000,000**

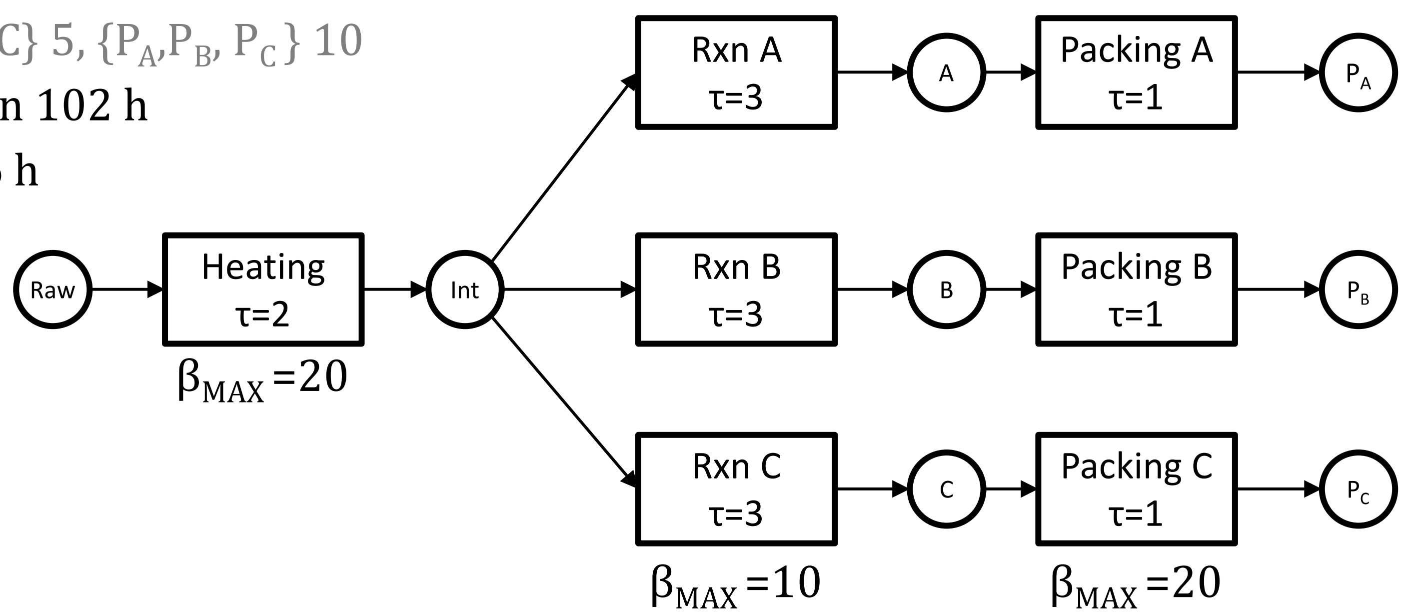
✓ Objective = E[Profit] – Expansion Cost = 1,600,000 – 100,000 = **$1,500,000**

Scheduling Problem

Example solved with (30 repetitions):

- Offline deterministic (nominal)
- Offline robust
- Offline stochastic programming (16 scenarios)
- Perfect information
- Online deterministic
- Online robust
- Online stochastic

- Facility: 2 heaters, 3 reactors, 2 packing lines; no storage
- Demand: nominal = 30 kg for P_A,P_B, P_C , simultaneously, at every 18 time steps; uniformly distributed with 50% uncertainty in demand magnitude; true demand known 12 time points ahead
- Prices: Raw 0, Int 1, {A,B,C} 5, {P_A,P_B, P_C} 10
- Total (simulation) horizon 102 h
- Rolling horizon length 36 h



Method Comparison

Closed-loop average costs

- Offline deterministic: $160K
- Offline robust: $350K
- Offline stochastic $110K
- Perfect information: $30K
- Online deterministic $85K
- Online robust: $80K
- Online stochastic: $70K

- **Adding feedback brings the most benefit**
- **The choice of model/method is secondary**

What Else Can We Do?

- **Stochastic optimization is (typically) computationally more expensive**
- **Can/should we solve a more *useful*, and equally expensive, problem?**

  e.g., Deterministic with longer planning horizon?

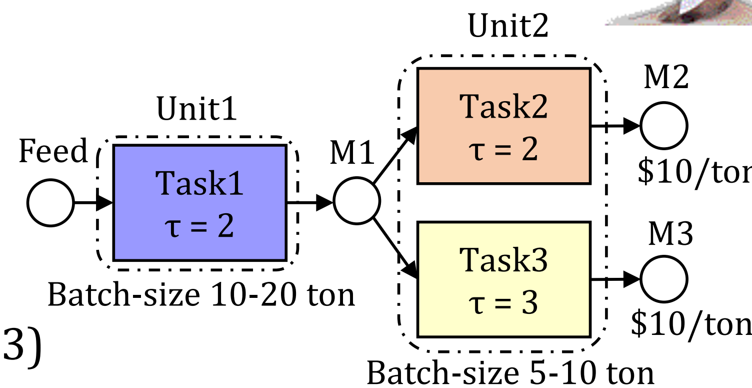
  Deterministic over a larger system?
- **Can/should we be doing anything else?**

  e.g., Solve a deterministic problem more frequently?

Uncertainty *vs.* New Information

Experiment #1

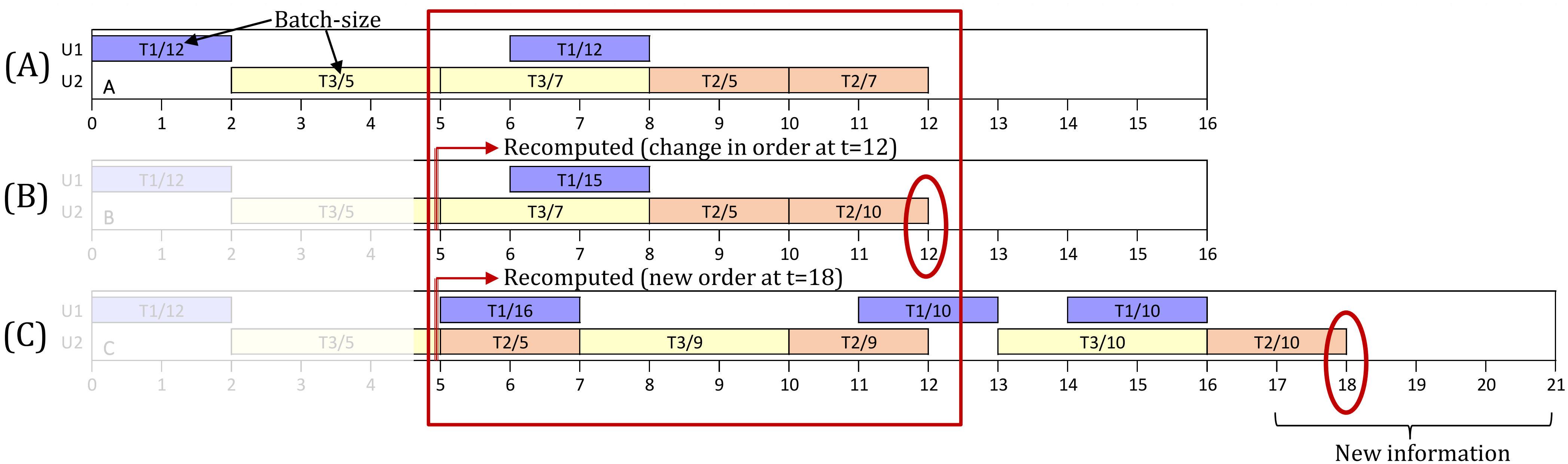
- Orders (12 tons M2, M3) due t=12, 18, 24, ...
- Use horizon H = 16 h



(A) Original schedule computed at t=0.

(B) Observation at t=5 → order due at t=12 changes (+3 tons M3)
Re-compute schedule - shrinking horizon

(C) At t=5 → order at t = 12 unchanged
Re-compute schedule - moving horizon



- (A, C) much more different than (A, B)

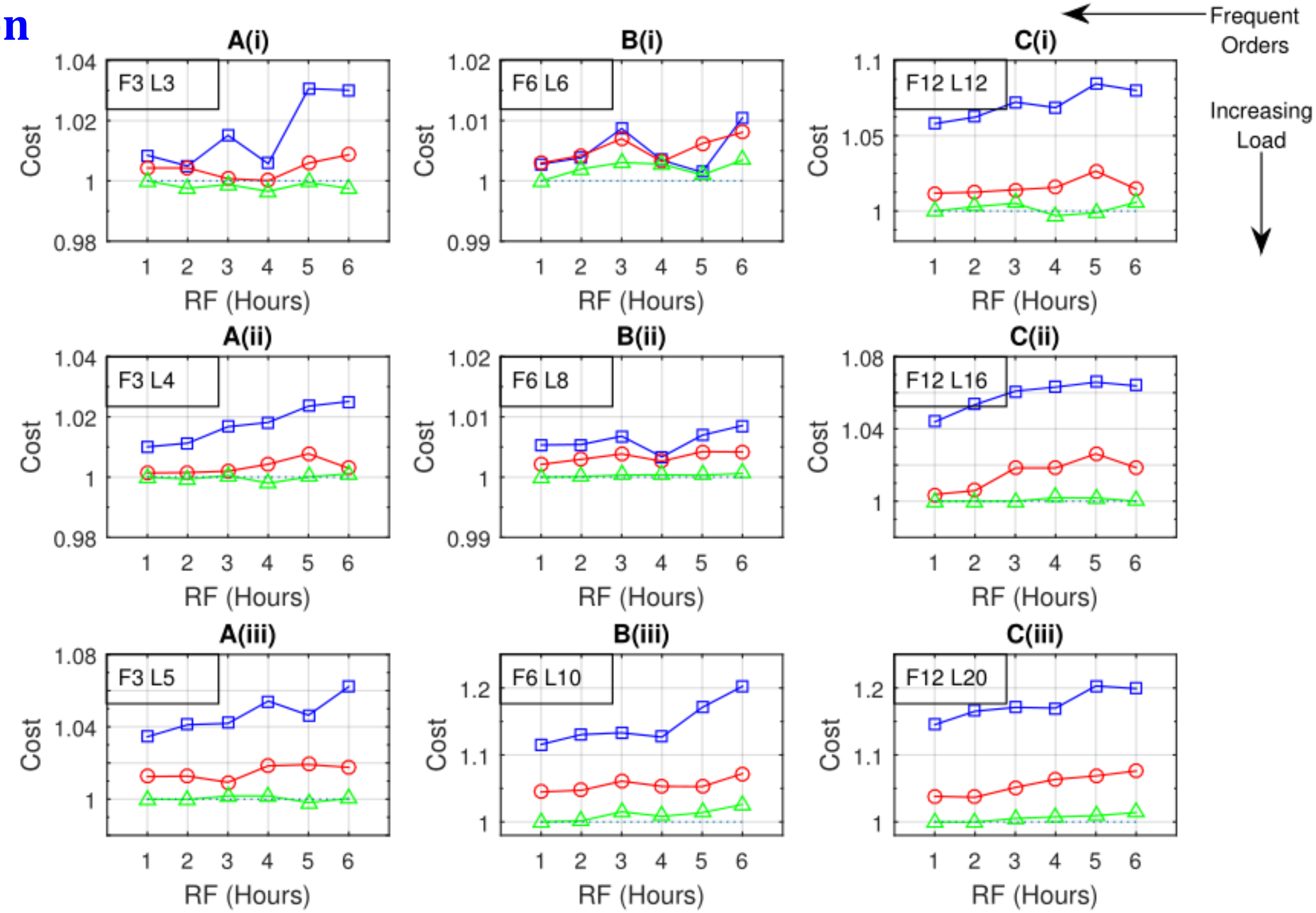
Accounting for new information can be more important than uncertainty

⇒ Use moving horizon long planning horizon

Rescheduling Frequency

Orders: every 3 hours | Orders: every 6 hours | Orders: every 12 hours

Cost minimization

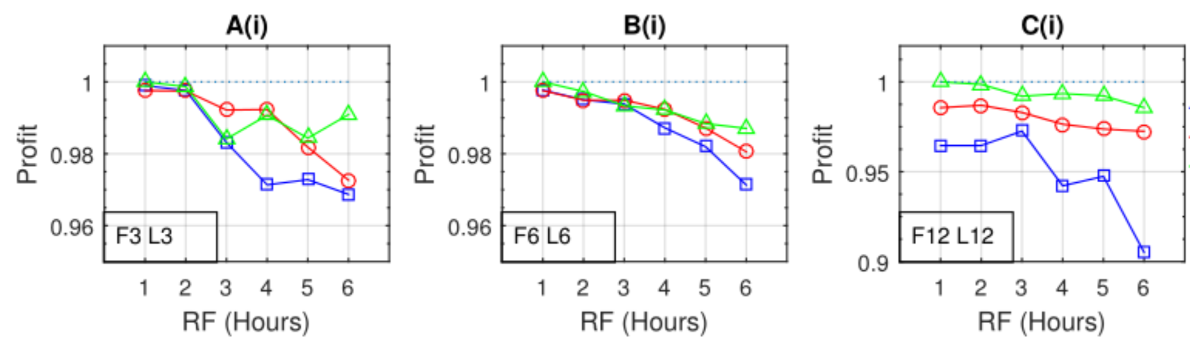


Load: 50% of capacity

Load: 75% of capacity

Load: 100% of capacity

Profit maximization

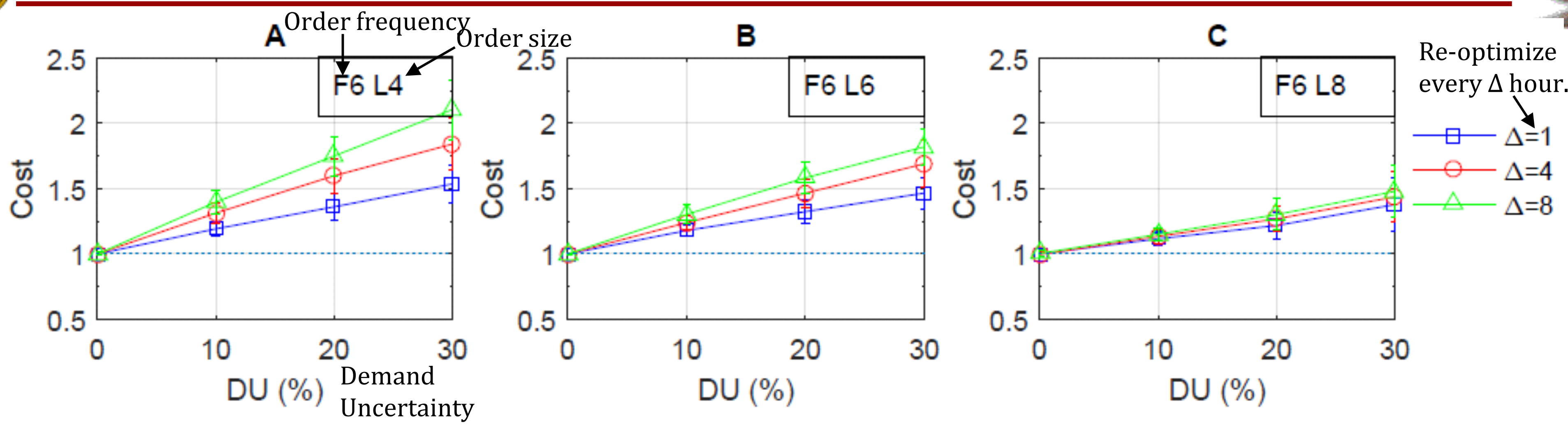


Load: 50% of capacity to meet demand, remaining capacity for extra sales

[1]*All open-loop solved to optimality.*

[2]*Each data point mean of 10 closed-loop runs (1 week each).*

Demand Uncertainty & Effect of Rescheduling Frequency



- Larger Demand Uncertainty (DU) increases closed-loop cost
- Frequent re-optimization is beneficial for “tackling” uncertainty (nominal model)

Design Framework for Online Scheduling Methods

- ❑ Operations are always reactive (or have a reactive element)
- ❑ Deterministic models can “handle” uncertainty through feedback & re-computation
- ❑ Re-optimize early and often (even if uncertainty is modeled)
- ❑ Question: what constraints and objective function weighs can we add to *obtain* good closed-loop solution?