# Multiscale Model-Based Process Systems Engineering

## **Progress and Challenges**

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### **Continuum Mesoscopic Equations**

Mesoscopic equation (for diffusion)

$$\begin{split} & \frac{\partial c_t = -\nabla \cdot \left(-\mu[c]\nabla(\delta\varphi[c]/\delta c)\right)}{\frac{\partial c}{\partial t} = \nabla \cdot \left\{ \mathrm{De}^{-\beta \mathrm{J}^* \mathrm{c}} \left[\nabla \mathrm{c} - \beta \mathrm{c}(1-\mathrm{c})\nabla \mathrm{J}^* \mathrm{c}\right] \right\}}{\mathrm{J}^* \mathrm{c} = \iint \mathrm{J}(|\mathrm{r} - \mathrm{r}'|)\mathrm{c}(\mathrm{r}')\mathrm{d}\mathrm{r}'} \end{split}$$

- Mesoscopic equations include essential physics
  - \* Intermolecular forces, microscopic mechanisms
  - \* Are exact when interactions are long ranged
- Amenable to systems tasks
- > Spectral methods for efficient, parallelizable calculations

PRL 85, 3898 (2000); PRL 84, 1511-1514 (2000); Chem. Eng. Sci. 58, 895 (2003)











## Realistic Nanomaterials Process Modeling: New Frontiers

> Product design and control at the nanoscopic scale

- > Emergent behavior and/or emergent properties
- > Combinatorial complexity
- > Validation, verification, and uncertainty
- Computer architectures





















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Open Questions
> Hierarchy of length and time scales
<ul> <li>Process model-based control impossible</li> </ul>
> Emergent behavior
<ul> <li>Sluggish dynamics, sensitivity to conditions</li> </ul>
<ul><li>Extremely high dimension (distributed parameter systems)</li></ul>
> Intrinsic stochastic fluctuations
> Poor controllability
* Control nanoscale features over several orders of magnitude
* A mere handful of manipulated variables at the macroscale
> Poor observability
<ul> <li>Measurements of features are unavailable online</li> </ul>







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## Likelihood Ratio Gradient Estimation

 A weighted average similar to importance sampling

$$\frac{\partial E(f)}{\partial \theta_j} = E\left(f\frac{\partial \ln p_v}{\partial \theta_j}\right)$$
$$\frac{\partial \ln p_v}{\partial \theta_j} = \sum_{i=1}^n \left[\frac{I(\mu_i, j)}{\theta_j} - \tau_i h_j(S_i)\right]$$
$$I(\mu_i) = \begin{cases} 1 & \text{if } \mu_i = j \end{cases}$$

$$I(\mu_i, j) = \begin{cases} 1 & \text{if } \mu_i = j \\ 0 & \text{if } \mu_i \neq j \end{cases}$$

- Calculate the entire gradient vector from *one* simulation
- Trivial code modification (record reaction fired and time)

McGill et al., *J. Comput. Phys.* **231**, 7170(2012)

 $\begin{array}{ll} I(\mu_{i\prime},j) & : \mbox{ indicator function} \\ \mu_i & : \mbox{ reaction fired at } i^{th} \mbox{ event} \\ \tau_i & : \mbox{ time step of } i^{th} \mbox{ event} \\ S_i & : \mbox{ state at } i^{th} \mbox{ event} \\ h_j(\cdot)_: & : \mbox{ related to propensity of } j^{th} \\ \mbox{ reaction} \\ p_v & : \mbox{ joint pdf for random} \\ variables \end{array}$ 





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### **Define Metrics**

- Difficult to use images as process outputs for control
   Translations and rotations: *morphologically equivalent* but *different images*
- Morphological information arises from image analysis
- > Define **metrics** that are *invariant to basic transforms* on the image

#### Metrics (9)

- ✓ Hexagonal order parameter
- ✓ Distance between islands
- ✓ Characteristic length
  - ✓ From spectral analysis of surface<sup>[1]</sup>
- ✓ Number of defects
- ✓ Minkowski measures<sup>[2]</sup>
  - ✓ Area fraction
    - ✓ Perimeter of nanodots
    - ✓ Euler characteristic (connectivity)



Abukhdeir and Vlachos, J. Comp. Phys. (2011); [2] Legland et al., Image Anal Sterol (2007)









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## **Open Questions**

- Uncertainty quantification
  - What data and at which scale to inject data?\*
  - > Beyond continuum models: e.g., Stochasticity
- Molecular structure and composition of catalyst
  - Optimize catalyst loading to explore bifunctionality
  - Extend to multicomponent catalysts
  - Account for crystal structure beyond core-shell
- Consider catalyst stability and catalyst dynamics
- Construct and search databases
  - Structure, composition, chemistry, reactor operating conditions

\* Prasad et al., Ind. Eng. Chem. Res. 47, 6555 (2008); 48, 5255 (2009)



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### **Conclusions/Outlook**

- Multiscale modeling of prototype models is mature
- Inverse engineering (optimization and control) at the nanoscale is challenging and offers opportunities
- Many interesting problems are either complex or emergent and their modeling is plagued by combinatorial complexity
- Hierarchical multiscale modeling is an approach coping with these issues
- Error quantification , code verification and validation, and uncertainty propagation are essential
- Parallelization holds promise for substantial speedup

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